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Electromagnetic waves in a partially ionized dusty plasma

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Abstract : The propagation of low-frequency electromagnetic waves in a partially ionized, self-gravitating magnetized dusty plasma has been theoretically studied. The electromagnetic waves having the wavelength of the order of a few *AU* and the frequency of the order of the order of 10^{-12} s^{-1} are found to be unstable due to the effect of the self-gravitational force acting on the dust grains. On the other hand, the electromagnetic waves having relatively high-frequency ($\sim 10^{-5}$ s^{-1}) are found to be damped due to the effect of the collisions of the plasma particles (namely, the electrons, the ions and the dust grains) with the background neutrals. The effects of the obliqueness of the wave propagation, the background neutral number density, the dust grain mass density, *etc.* are also observed to significantly modify the dispersion properties of such obliquely propagating electromagnetic waves. The present investigation may be of relevance to the formation fo stars in galaxies *via* the fragmentaion of partially ionized dusty plasma systems.

Keywords : Dusty plasma, electromagnetic waves, dispersion properties, gravitational instability.

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1. Introduction

Partially ionized dusty plasmas [1–3] are very common in most of the astrophysical objects and space environments, such as, interstellar molecular clouds, protostellar disks, asteroid zones, planetary atmospheres, interstellar media, cometary tails, nebula, earth's environments, *etc.* The physics of the interaction of partially ionized dusty plasmas with the fields of propagating electrostatic/electromagnetic waves has nowadays become a challenging topic of research because of its vital role in understanding the dynamics and fragmentation of interstellar molecular clouds, star formation, galactic structure and its evolution, magnetic reconnection, evolution of the magnetic field in such different astrophysical dusty plasma systems, *etc.* [4–12]. Jeans [13] first predicted the instability of self-gravitating large gas clouds. Edington [14] investigated the gravito-electromagnetic coupling of an atomic plasma at local thermal equilibrium in the context of star formation. Chandrasekhar [15] and Friedman and Polyachenko [16] have comprehensively investigated the gravitational (or Jeans) instability in self-gravitating fluids and plasmas. These pioneering works [13–16] have later been extended by a large number of authors [17–32].

Zweibel [17], Balsara [18], Bulanov and Sakai [19], *etc.* have studied waves and instabilities in a self-gravitating, partially ionized magnetoplasma by ignoring the presence or the dynamics of the dust particles or the plasma effects. However, the dynamics or even the presence of charged dust particles in the interstellar molecular clouds (which is, in fact, a very weakly ionized dusty plasma with a significant fraction of neutrals) may affect the gravitational contraction process which causes the collapse of the large interstellar clouds. The dust particles by the process of coagulation, turn out to be a dust ball which serves as a core and around such cores, the gaseous components of the cloud can collapse. Therefore, the presence or the dynamics of charged dust grains essentially plays an important role in star formation *via* self-gravitational instability [6–9]. On the other hand, self-gravitational instabilities in dusty magnetoplasmas have been investigated by a number of authors, for examples, Shukla and Rahman *et al* [20], Verheest *et al* [21], Mahanta [22], Mamun [23], Salimullah

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and Shukla [24], *etc.* They [20–24] have completely ignored the presence of the background neutral atoms, *viz.* neglected the effects of collisions of the plasma particles (namely, the electrons, the ions, and the dust grains) with the background neutrals which, of course, play a very significant role in instability/damping of the wave propagation in such weakly ionized, self-gravitating dusty magnetoplasma systems [25–31].

Recently, we [32] have considered a partially ionized, two fluid dusty magnetoplasma (where all electrons are assumed to be depleted onto the surface of the extremely massive dust grains) and have studied electromagnetic modes propagating perpendicualr to the external magnetic field. In the present investigation, we consider a more practical situation, a four component (electron, ion, dust and incompressible neutral fluids) self-gravitating dusty magnetoplasma and study the obliquely propagating low-frequency electromagnetic waves whose frequency (ω) lies between the dust-cyclotron (ω_{cd}) and ion-cyclotron (ω_{ci}) frequencies, *i.e.* $\omega_{cd} < \omega < \omega_{ci}$.

The manuscript is organized as follows. We present the governing equations in Section 2. We derive the dispersion relation and numerically investigate the dispersion properties of the obliquely propagating low-frequency electromagnetic waves and associated instability/damping in Section 3. A brief discussion is finally presented in Section 4.

2. Governing equations

We consider a weakly ionized, self-gravitating, four component dusty magnetoplasma, consisting of electron, ion and negatively charged dust, and incompressible neutral fluids. The dusty plasma is assumed to be qausineutral, *i.e.*, $N_i = N_e + Z_d N_d$, where N_i, N_e and N_d are the ion (single charged), the electron and the dusty particle number density, respectively and Z_d is the number of electronic charge residing onto the surface of the dust grain. We also suppose that the dusty plasma is immersed in a uniform external magneitc field $\boldsymbol{B}_0 = \hat{z}B_0$, where $\hat{z}$ is the unit vector along the z-axis. The basic equations governing the macroscopic state of such a weakly ionized, self-gravitating, dusty magnetoplasma are

$$\frac{\partial Ni}{\partial t} + \nabla\cdot(N_i \boldsymbol{u}_i) = 0 \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}_i\cdot\nabla\right)\boldsymbol{u}_i = \frac{e}{m_i}\left[\boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_i\times(\boldsymbol{B}_0+\boldsymbol{B})\right] - \frac{\gamma_i T_i}{N_i m_i}\nabla N_i - \nu_{in}\boldsymbol{u}_i, \tag{2}$$

$$\frac{\partial N_e}{\partial t} + \nabla\cdot(N_e \boldsymbol{u}_e) = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}_e\cdot\nabla\right)\boldsymbol{u}_e = -\frac{e}{m_e}\left[\boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_e\times(\boldsymbol{B}_0+\boldsymbol{B})\right] - \frac{\gamma_e T_e}{N_e m_e}\nabla N_e - \nu_{en}\boldsymbol{u}_e, \tag{4}$$

$$\frac{\partial N_d}{\partial t} + \nabla\cdot(N_d \boldsymbol{u}_d) = 0, \tag{5}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}_d\cdot\nabla\right)\boldsymbol{u}_d = -\frac{Z_d e}{m_d}\left[\boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_d\times(\boldsymbol{B}_0+\boldsymbol{B})\right] - \frac{\gamma_d T_d}{N_d m_d}\nabla N_d - \nabla\Psi_g - \nu_{dn}\boldsymbol{u}_d, \tag{6}$$

$$\nabla^2\Psi_g = 4\pi G m_d N_d, \tag{7}$$

$$\nabla\times\boldsymbol{B} = \frac{4\pi e}{c}(N_i\boldsymbol{u}_i - N_e\boldsymbol{u}_e - Z_d N_d\boldsymbol{u}_d), \tag{8}$$

$$\boldsymbol{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t}, \tag{9}$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A}, \tag{10}$$

where $\boldsymbol{u}_s$ is the fluid velocity of the species s ($s = e$ for electrons, $s = i$ for ions, and $s = d$ for dust), m_s is the mass, γ_s is the adiabatic index, and T_s is the temperature in units of energy. Furthermore, $\nu_{sn} = \sigma_{sn}n_n(T_s/m_s)^{1/2}$ is the collision frequency of the species s with neutral atoms, where σ_{sn} is the collision cross section which is typically $\sim 5\times10^{-15}$ cm^2 and n_n is the background neutral number density. $\boldsymbol{E}$, $\boldsymbol{B}$ and Ψ_g are the wave electric field, the wave magnetic field, and the self-gravitational potential, respectively, ϕ is the scalar potential, $\boldsymbol{A}$ is the vector potential, e is the electronic charge, G is the universal gravitational constant, and c is the speed of light in vacuum. We have neglected the contribution of collisions between the plasma particles themselves since a very weakly ionized plasma (*i.e.*, $n_n \ll n_{i0}, n_{d0}, n_{e0}$, where n_{s0} is the background plasma particle number density, typically, in molecualr clouds [27–31] $n_n \sim 10^4$ cm^{-3}, $n_{i0} \sim 10^{-3}$ cm^{-3}, $n_{d0} \sim 4.5\times10^{-7}$ cm^{-3}, $Z_d = 2.0\times10^3$, $n_{e0} \sim 10^{-4}$ cm^{-3}) are considered. We have also neglected the contributions due to the displacement current and the gravitational forces acting on electrons and ions.

3. Dispersion properties

To study the dispersion properties of the obliquely propagating electromagnetic waves under consideration, in a weakly ionized, self-gravitating dusty magnetoplasma, we shall carry out a normal mode analysis. Thus, we first express our dependent varibles in terms of their equilibrium and perturbed parts as $N_s = n_{s0} + n_s$, $u_s = 0 + u_s$, $E = 0 + E$, $B = 0 + B$, $\Psi_g = 0 + \Psi_g$, $\phi = 0 + \phi$ and $A = 0 + A$, where $n_s << n_{s0}$. Thus, our basic eqs. (1)–(10) are linearized as

$$\frac{\partial n_i}{\partial t} + n_{i0}\,\nabla . \boldsymbol{u}_i = 0, \tag{11}$$

$$\frac{\partial \boldsymbol{u}_i}{\partial t} = \frac{e}{m_i}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_i \times \boldsymbol{B}_0\right) - \frac{T_i}{n_{i0} m_i}\nabla n_i - \nu_{in}\boldsymbol{u}_i, \tag{12}$$

$$\frac{\partial n_e}{\partial t} + n_{e0}\,\nabla . \boldsymbol{u}_e = 0, \tag{13}$$

$$\frac{\partial \boldsymbol{u}_e}{\partial t} = -\frac{e}{m_e}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_e \times \boldsymbol{B}_0\right) - \frac{T_e}{n_{e0} m_e}\nabla n_e - \nu_{en}\boldsymbol{u}_e, \tag{14}$$

$$\frac{\partial n_d}{\partial t} + n_{d0}\,\nabla . \boldsymbol{u}_d = 0, \tag{15}$$

$$\frac{\partial \boldsymbol{u}_d}{\partial t} = \frac{Z_d e}{m_d}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{u}_d \times \boldsymbol{B}_0\right) - \frac{T_d}{n_{d0} m_d}$$

$$\nabla N_d - \nabla \Psi_g - \nu_{dn}\boldsymbol{u}_d, \tag{16}$$

$$\nabla^2 \Psi_g = 4\pi G m_d n_d . \tag{17}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi e}{c} + \left(n_{i0}\boldsymbol{u}_i - n_{e0}\boldsymbol{u}_e - Z_d n_{d0}\boldsymbol{u}_d\right) \tag{18}$$

$$\boldsymbol{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t}, \tag{19}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}. \tag{20}$$

Now, using eqs. (11)–(20), we can eliminate $\boldsymbol{E}$, $\boldsymbol{B}$, n_e, n_i, Ψ_g, $\boldsymbol{u}_e$, $\boldsymbol{u}_i$, $\boldsymbol{u}_d$, and for $\omega_{cd} << \omega << \omega_{ce}$ (where $\omega_{ce} = eB_0/m_e c$), we can reduce eqs. (11)–(20) to a set of three coupled equations

$$\left(D_t^2 + \omega_{ci}^2\right)\left(Z_d\frac{\partial n_d}{\partial t} - \frac{c}{4\pi e}\frac{\partial}{\partial z}\nabla_\perp^2 A_z\right)$$

$$-\frac{n_{i0}e}{m_i}D_t\nabla_\perp^2 \times \phi = 0, \tag{21}$$

$$\left[\left(1 - \lambda_e^2\nabla_\perp^2\right)\frac{\partial^2}{\partial t^2} - \nu_{en}\lambda_e^2\nabla_\perp^2\frac{\partial}{\partial t} + c^2\lambda_{De}^2\nabla_\perp^2\frac{\partial^2}{\partial z^2}\right]A_z$$

$$+ c\frac{\partial^2\phi}{\partial z\partial t} = 0, \tag{22}$$

$$\left[\left(\frac{\partial}{\partial t} + \nu_{dn}\right)\frac{\partial}{\partial t} - \omega_{Jd}^2\right]n_d + \frac{Z_d n_{d0} e}{m_d}\nabla^2\phi +$$

$$\frac{Z_d n_{d0} e}{m_d c}\frac{\partial^2 A_z}{\partial z\partial t} = 0, \tag{23}$$

where $D_t = \partial/\partial t + \nu_{in}$, $\lambda_e = c/\omega_{pe}$ is the electron skin depth, $\lambda_{De} = (Te/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye length, $\omega_{Jd} = (4\pi G\rho_d)^{1/2}$ is the dust Jeans frequency, $\rho_d = m_d n_{d0}$ is the background dust mass density, $\omega_{ci} = eB_0/m_i c$ is the ion gyrofrequency, and $\omega_{pe} = (4\pi n_{e0}e^2/m_e)^{1/2}$ is the electron plasma frequency. The subscript $\perp(z)$ on a quantity represents the corresponding quantity perpendicular (parallel) to the external magnetic field $\boldsymbol{B}_0 = \hat{z}B_0$. Furthermore, in deriving eqs. (21)–(23), we have assumed that (i) the current along the z-direction is mainly due to the electrons and A is along the z-direction, *i.e.*, $\boldsymbol{A} = \hat{z}A_z$, (ii) $\nabla . \boldsymbol{u}_{e\perp} << \partial u_{ez}/\partial z$ and $\nabla . \boldsymbol{u}_{i\perp} >> \partial u_{iz}/\partial z$ and (iii) $|\partial/\partial t|$, $\nu_{in} >> \upsilon_{ti}|\nabla|$, and $|\partial/\partial t|$, $\nu_{dn} >>> \upsilon_{td}|\nabla|$, where $\upsilon_{ti} = (\gamma_i T_i/m_i)^{1/2}$ and $\upsilon_{td} = (\gamma_d T_d/m_d)^{1/2}$ are the ion-thermal speed and the dust-thermal speed, respectively.

We now assume that all the perturbed quantities are proportional to $\exp(-i\omega t + \boldsymbol{k}.\boldsymbol{r})$.

Thus from eqs. [21–23], we can obtain a general disperssion relation.

$$\alpha_1\omega_{pd}^2\left(k_z^2 - \alpha_2 k^2\right) - \alpha_1\alpha_3 c^2 k_z^2 k_\perp^2$$

$$-\alpha_2\alpha_3\omega_{pi}^2\left(1 + i\frac{\nu_{in}}{\omega}\right)k_\perp^2 = 0, \tag{24}$$

where

$$\alpha_1 = \left(1 + i\frac{\nu_{in}}{\omega}\right)^2 - \frac{\omega_{ci}^2}{\omega^2}, \tag{25}$$

$$\alpha_2 = 1 + k_\perp^2\lambda_e^2\left(1 + i\frac{\nu_{en}}{\omega}\right) - \frac{c^2k_z^2}{\omega^2}k_\perp^2\lambda_{De}^2, \tag{26}$$

$$\alpha_3 = 1 + i\frac{\nu_{dn}}{\omega} + \frac{\omega_{Jd}^2}{\omega^2}. \tag{27}$$

Here, $\omega_{pi} = (4\pi n_{i0}e^2/m_i)^{1/2}$ is the ion plasma frequency, and $\omega_{pd} = (4\pi n_{d0}Z_d^2e^2/m_d)^{1/2}$ is the dust plasma frequency. We note that $k_z = k\cos\theta$ and $k_\perp = k\sin\theta$, where θ is the angle between the directions of the external magnetic field and the wave propagation.

Eq. (24) is our new dispersion relation for obliquely propagating electromagnetic waves modified by the combined effects of the self-gravitational force, the drag forces (*i.e.*, collisions of plasma particles with the background neutrals), the obliqueness of the wave propagation, *etc.* in a weakly ionized, self-gravitating dusty magnetoplasma. We now study numerically how these effects modify the dispersion properties and associated instability/damping of the electromagnetic waves under consideration. In our numerical analysis of the general dispersion relation (24), we have chosen the plasma parameters which are typical for interstellar molecualr clouds [27–31] : $n_n = 10^4$ cm^{-3}, $n_i = 10^{-3}$ cm^{-3}, $n_d = 4.5 \times 10^{-7}$ cm^{-3}, $Z_d = 2.0 \times 10^3$, $m_d = 4.0 \times 10^{-12}$ gm, $B_0 = 10$ μG, $T_e = 10^4$ K, $T_i = 10^3$ K, $T_d = 10$ K and $\theta = 45°$. The numerical resuslts are displayed in Figures 1–3.

Figures 1 and 2 show the dispersion properties of the electromagnetic waves having the wavelength of the order of 1 *AU* and the frequency of the order of 10^{-4} s^{-1}. Figure 1 shows how the effect of the background neutral number density (n_n) modifies their dispersion properties. It is seen from Figure 1 that the electromagnetic waves are damped due to the collisions of the plasma particles with the background neutrals. It is also found that as we increase n_n (keeping the other parameters constant), the real frequency of the mode decreses, whereas the damping rate increases. Figure 2 shows how the effect of the obliqueness (θ) of the propagation of these damped electromagnetic waves modify their dispersion properties. It is observed

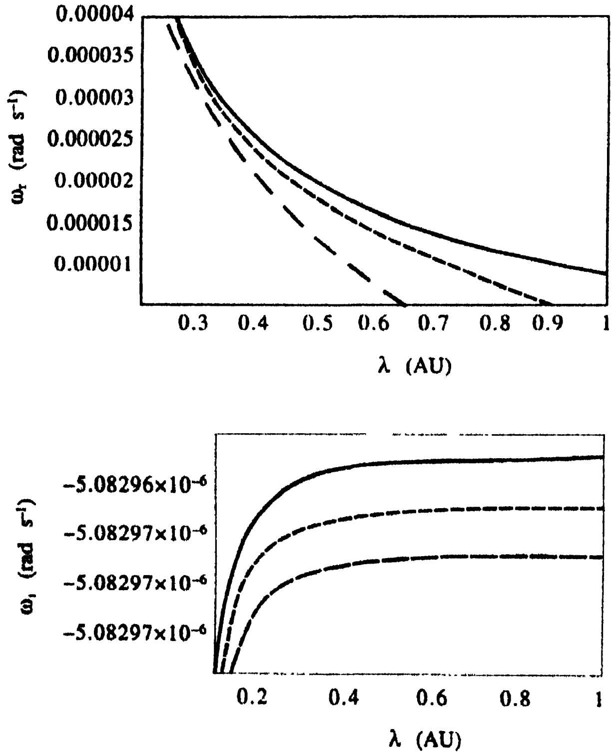

Figure 1. Effects of n_n on the dispersion properties of the damped electromagnetic waves. The upper plot where n_n = 10000 cm^{-3} (solid curve), n_n = 19000 cm^{-3} (dotted curve) and n_n = 30000 cm^{-3} (dash curve), is for the real frequency ω_r, whereas the lower plot where n_n = 10000 (solid curve), n_n = 10001 cm^{-3} (dotted curve) and n_n = 10002 (dash curve), is for the damping rate ω_i. The other parameters are given in the text.

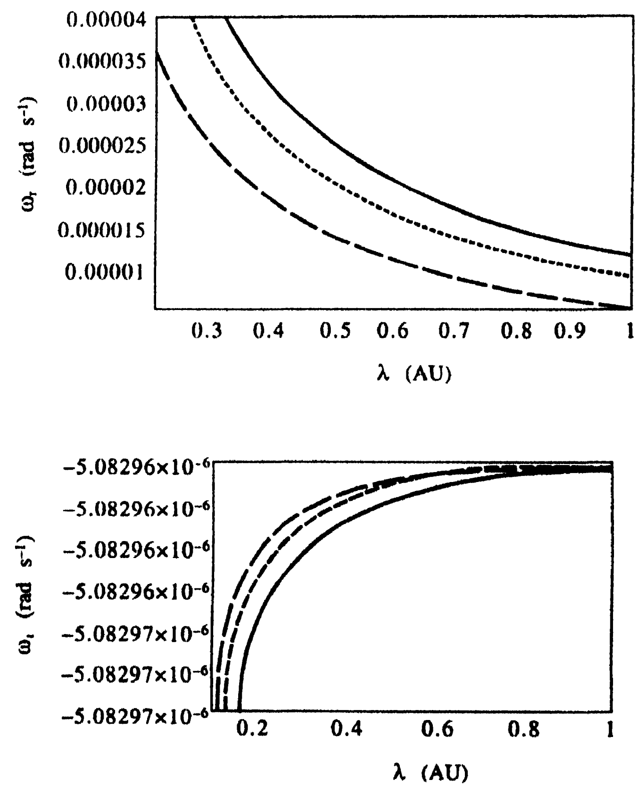

Figure 2. Effects θ on the dispersion properties of the damped electromagnetic waves for $\theta = 30°$ (solid curve), $\theta = 45°$ (dotted curve) and $\theta = 60°$ (dash curve). The upper plot is for the real frequency ω_r, whereas the lower plot is for the damping rate ω_i. The other parameters are given in the text.

from Figure 2 that as we increase θ (keeping the other parameters constant), both the real frequency and the damping rate of these electromagnetic waves increase.

Figure 3 shows the dispersion properties of the electromagnetic waves having the wavelength of the order of a few AU and the frequency of the order of 10^{-11} s^{-1}. It is found here that when the wavelength of the electromagnetic waves exceeds a critical value (depending mainly on the value of the dust mass density), the waves suffer the gravitational instability (with the growth rate of the order of 10^{-12} s^{-1}) due to the gravitational force acting on the massive dust grains. It is bovious here that as we increase the dust mass density (ρ_d), the critical wavelength (the wavelength for which the electromagnetic waves suffer the gravitational instability) is shifted to lower values (*cf.* the lower plot). It is also found that as we increase the dust mass density (ρ_d), the real frequency of the mode decreases whereas the growth rate of the instability increases.

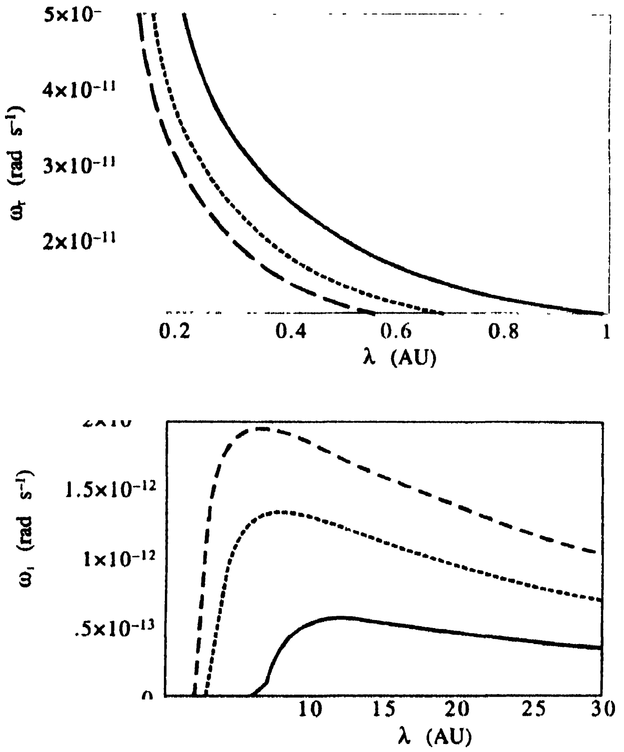

Figure 3. Effects of ρ_d on the dispersion properties of the unstable electromagnetic waves for $\rho_d = 5 \times 10^{-18}$ gm cm^{-3} (solid curve), $\rho_d = 1.0 \times 10^{-17}$ gm cm^{-3} (dotted curve) and $\rho_d = 1.5 \times 10^{-17}$ gm cm^{-3} (dash curve). The upper plot is for the real frequency ω_r, whereas the lower plot is for the growth rate ω_i of the gravitational instability. The other parameters are given in the text.

4. Discussion

We have considered the propagation of low-frequency electromagnetic waves in a multicomponent, self-gravitating dusty magnetoplasma whose constituents are electrons, ions, dust grains and neutrals. We have focused on obliquely propagating ultra-low-frequency electromagnetic waves whose frequencies lie between the dust-cyclotron and ion-cyclotron frequencies *i.e.* $\omega_{cd} < \omega < \omega_{ci}$. Taking plasma parameters that are relevant to interstellar molecular clouds, we have shown that the electromagnetic waves having the wavelength of the order of a few AU and the frequency of the order of 10^{-12} s^{-1} are unstable due to the effect of the self-gravitational force acting on the dust grains, whereas those having relatively high frequency of the order of $\sim 10^{-5}$ s^{-1} are damped due to the effects of collisions of the plasma particles (namely, the electrons, the ions and the dust grains) with the background neutral atoms. We have also found that as we increase n_n (keeping the other parameters constant), the real frequency of the mode decreases, whereas the damping rate increases.

The results of our numerical analysis reveal that when wavelength of the electromagnetic waves exceeds a critical value (depending on the value of the dust mass density), the waves suffer the gravitational instability (with the growth rate of the order of 10^{-12} s^{-1}) due the gravitational force acting on the massive dust grains. It is obvious here that as we increase the dust mass density, the critical wavelength (the wavelength for which the electromagnetic waves suffer the gravitational instability) is shifted to lower values. It is also shown that as we increase the dust mass density, the real frequency of the mode decreases whereas the growth rate of the instability increases. It may be stressed here that the present investigation may be of relevance to the formation of stars in galaxies *via* the fragmentation of weakly ionized dusty plasma systems like interstellar molecualr clouds.

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